

# Being Rational or Aggressive? A Revisit to Dunbar's Number in Online Social Networks

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## ABSTRACT

Recent years have witnessed the explosion of online social networks (OSNs). They provide powerful IT-innovations for online social activities such as organizing contacts, publishing contents, and sharing interests between friends who may never meet before. As more and more people become the active users of online social networks, one may ponder questions such as: (1) Do OSNs indeed improve our sociability? (2) To what extent can we expand our offline social spectrum in OSNs? (3) Can we identify some interesting user behaviors in OSNs? Our work in this paper just aims to answer these interesting questions. To this end, we pay a revisit to the well-known Dunbar's number in online social networks. Our main research contributions are as follows. First, to our best knowledge, our work is the first one that systematically validates the existence of the online Dunbar's number in the range of [200,300]. To reach this, we combine using local-structure analysis and user-interaction analysis for extensive real-world OSNs. Second, we divide OSNs users into two categories: rational and aggressive, and find that rational users intend to develop close and reciprocated relationships, whereas aggressive users have no consistent behaviors. Third, we build a simple model to capture the constraints of time and cognition that affect the evolution of online social networks. Finally, we show the potential use of our findings in viral marketing and privacy management in online social networks.

## Categories and Subject Descriptors

J.4 [Computer Applications]: Social and behavioral sciences

## General Terms

Human Factors, Measurement

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## Keywords

Online social networks, Dunbar's Number, User Behavior, Network Evolution Modeling, Viral Marketing, Privacy

## 1. INTRODUCTION

In this day and age, the online social sites, like Facebook<sup>1</sup>, Livejournal<sup>2</sup>, MySpace<sup>3</sup> and etc., provide people with a new powerful means to communicate and interact with each other. Through these sites, users can share blogs, photos and current statuses. They can consolidate friendships in the real-world by exchanging information online and establish new virtual friendships with others in the same site. It is these sites that lead to the formation of a new kind of social network, which is called the online social network. Indeed, with the thorough development of online social sites in the recent decade, the online social network has become an essential part of our daily life and is changing our social behaviors potentially. At the same time, different from the traditional real-world social network, the electric communication data of the online social network is relatively easy to collect[5]. Besides, compared with the real-world social network, its scale is huge. So it is reasonable to conjecture that this new form of network would give many inspirations to the previous recognition of the social networks.

Through coupling the number of friendships and the size of the neocortex in primates, Dunbar found humankind can only maintain as many as 150 friendships effectively[6]. And the number 150 is then called the magic number in social networks. In our experience of using online social networks, we can easily find that some users have extremely large number of friends, much more than 150, while others keep only an averaged level of friends. We believe it is the online mechanisms that facilitate the formation of high-degree nodes, since friends making is so convenient that only requires an invitation to be added as a friend and an acceptance. Therefore we may cast doubts on whether online social networks deviate from the constraint of Dunbar's number. To verify the doubts, it is necessary to investigate the following questions:

- Does there still exist a magic number in online social networks as Dunbar's number in the real-world social networks?

<sup>1</sup><http://www.facebook.com>

<sup>2</sup><http://www.livejournal.com>

<sup>3</sup><http://www.myspace.com>

- If it exists, what’s its value and how it generates?
- How it changes things?

In this paper, we aim to answer the above questions through the analysis on several datasets coming from some online social sites. By observing many local measures, we conclude that there exists a new magic number pervasively, which is in the range of 200 and 300, greater than 150 found previously in real-world social networks. We also validate this by investigating the traces of interaction between many users in online social sites. We find through our observations that although the online social sites provide us many easier ways to maintain online friendships effectively, there is still an upper limit on the number of substantial and meaningful friends. Furthermore, we believe that users can be distinguished by the magic number and they exhibit different behaviors and attitudes, respectively.

Given the fact that many current models cannot interpret these phenomena, we present a new simple model to interpret how the magic number in online social networks generates. Finally, we think this number is insightful to guide the viral marketing strategy and user privacy management. For instance, hub nodes may not be effective choice in viral marketing, and certain users call for a detailed privacy setting mechanism.

The rest of the paper is organized as follows. In Section 2, some related works will be introduced. In Section 3, we will define some local and global measures used in the following analysis. Our observations and findings will be depicted in Section 4. In Section 5, we present a new model to interpret the new upper limit existing pervasively in online social networks. We also give a talk about the business insights about the new magic number in Section 6. Finally, we conclude this paper in Section 7.

## 2. RELATED WORK

Our study is related to the work in three areas: the phenomenon of Dunbar’s number, measurement analysis of online social networks and social network modeling.

### 2.1 The phenomenon of Dunbar’s number

By investigating the relationship between neocortex size and group size in primates, Dunbar[6] predicted the number of group size in human beings was 150, which was notable as Dunbar’s number. According to him, human beings can only maintain a small fraction of relationships within the circle of Dunbar’s number, and other relationships beyond that circle are not reciprocated or personalized. Dunbar’s number targets on real-world social networks when first put forward. However, recent works in online social networks have displayed similar interesting observations [1, 7]. Roberts et al. pointed out that time spent using social media, including online social sites, was not associated with larger offline networks [20]. Potential time and cognitive constraints were also considered in their work. Other work of online social networks related to Dunbar’s number will be further discussed in Section 2.2.

### 2.2 Measurement analysis of online social networks

Recently, researchers have done intensive study in online social networks. They measured the property of online social networks from different perspectives. Phenomena such

as small world, power-law, high clustering, assortativity have been observed in different social sites, which are believed to be the common properties of online social networks. Ahn et al. studied the largest online social networks Cyworld in South Korea[1]. They experimented on the whole data of Cyworld and discovered some unique characters of this site. They found an interesting phenomenon that most user connections were not active and attributed it to Dunbar’s number. Mislove et al. used data from Flickr, YouTube, LiveJournal and Orkut, conducting measurement analysis in a large scale[17]. They incorporated various complex network measurements such as degree distribution, clustering coefficients, degree correlations, connected core etc. in the research. Golder et al. analyzed Facebook users in North American colleges or universities[7]. Their results on degree distribution showed that the number of people who have few hundreds of friends remained stable, but it started to drop sharply once the friends number exceeded 250, which also coincided with Dunbar’s number.

## 2.3 Social networks modeling

Although Small-World[24] and BA[2] network models lay a foundation for the complex network, these two models cannot explain all the phenomena of different types of real networks. As for social networks, multiple models have been proposed to fit their particular properties. Holme and Kim added a “triad formation step” beyond BA model(HK), *i.e.* establishing edges between neighbors of a node[11]. David-son et al. imported a similar process in their DEB model[4]. This process in fact corresponds to the real-world social network situation, as people are easily introduced to meet friends of their friends and build up connections. Both networks generated from HK and DEB have power-law and high clustering properties. Jin et al. carefully studied the principles of social network formation, and proposed a social network model (JGN)[13]. JGN also considered the influence of mutual friends, cost of friend maintenances and the maximum connections. JGN and HK are the two early models to generate social networks, and they both grasp the core principle to generate networks-“transitivity”. Thus many successive models inherit the idea of “transitivity” from them. The models mentioned above mainly focus on the real-world social networks, while online factors are not taken into account. Although online social networks have gained popularity in recent research, the work to model this kind of network is still quite insufficient. Yuta et al. pointed out that the cost of online friend maintenance was much lower, and they extended CCN[22] by adding a process of random linkage to form a new model CCNR[25]. Bonato et al. also adopted transitivity in their model ILT[3].

In summary, almost all the social network models, no matter online or offline, adopt the rule of transitivity in various forms. And the networks generated by these models commonly have the feature of high clustering due to this fact.

## 3. PRELIMINARIES

In this section, we depict definitions of some critical global and local measures which we would use in the following sections.

An online social network can be intuitively modeled as a graph  $G(V, E)$ , where  $V$  is the set of users and  $E$  is the set of ties. For the reason that establishing a new tie usually needs mutual permission in online social sites,  $G$  is undirected.

Generally, the number of a node's friendships can be defined as its degree. The averaged degree of the network can be defined as

$$\langle k \rangle = \frac{2|E|}{|V|}. \quad (1)$$

$k_{max}$  is the maximum degree among all the nodes and  $k_{min}$  is the minimum degree.  $p(k)$  is the degree distribution of the graph and for online social networks, it is always power-law. Usually, the complementary cumulative distribution function (*CCDF*) is used to characterize this.

Clustering coefficient of a node is used to characterize how closely its neighbors are connected. It can be defined as

$$C_i = \frac{2|E_i|}{k_i(k_i - 1)}, \quad (2)$$

where  $E_i$  is the set of ties between  $i$ 's neighbors and  $k_i$  is the degree of  $i$ . For the case of  $k_i = 1$ , we set  $C_i = 0$  in this paper. Then the averaged clustering coefficient of the nodes with degree  $k$  can be defined as

$$C(k) = \frac{\sum_{\{i \in V | k_i = k\}} C_i}{|\{i \in V | k_i = k\}|}. \quad (3)$$

The averaged clustering coefficient of the network can be defined as

$$C = \frac{\sum_{\{i \in V\}} C_i}{|V|}. \quad (4)$$

The averaged clustering coefficient of the social network is always higher than the technical network.

The averaged degree of a node's neighbors, denoted as  $k_{nn}$ , is always used to depict the assortativity of the network. If the network is disassortative, the nodes with low degree is preferentially connected to ones with high degrees, then  $k_{nn}$  will decrease with the increment of the degree. Contrarily, the nodes will be connected to those with similar degrees when the network is assortative. The social network is usually thought to be assortative. Here we define  $i$ 's  $k_{nn}$  as

$$k_{nn}^i = \frac{\sum_{j \in \{i's \text{ neighbors}\}} k_j}{k_i}. \quad (5)$$

Similarly, the averaged  $k_{nn}$  of the nodes with degree  $k$  can be defined as

$$k_{nn}(k) = \frac{\sum_{\{i \in V | k_i = k\}} k_{nn}^i}{|\{i \in V | k_i = k\}|}. \quad (6)$$

It can be divided by  $k_{max}$  to be normalized.

K-shell (k-core) index, denoted as  $k_s$ , is usually used to characterize how far is a node away from the core of the network. For instance, greater value of  $k_s$  means the node is closer to the core. It can be obtained through the following method [14]. First, remove all the nodes with degree  $k = 1$ . After this stage of pruning, there may appear new nodes with  $k = 1$ . Then keep on pruning these nodes, as well, until all nodes with degree  $k = 1$  are removed.  $k_s$  of the removed nodes will be set to 1. Next, we repeat the pruning process in a similar way for the nodes with degree  $k = 2$  and subsequently for higher values of  $k$  until all nodes are removed. In [14], it is found that in many networks, including online social networks, high-degree nodes may have low  $k_s$ , indicating that those nodes were at the periphery of the network. The averaged k-shell index of the nodes with

degree  $k$  can be defined as

$$k_s(k) = \frac{\sum_{\{i \in V | k_i = k\}} k_s^i}{|\{i \in V | k_i = k\}|}, \quad (7)$$

where  $k_s^i$  is the k-shell index of  $i$ .

The strength of a tie between two nodes in a social network is usually defined as the overlap of their friends [8, 12]. It means the more common friends they share, the more familiar they would be. In online social networks, sharing more common friends usually means they are geographically close to each other, or share the same profiles, or interact more frequently online. Online friends with a big value of tie strength may have a higher probability to be friends in offline social networks. We define the strength of tie between  $i$  and  $j$  as

$$w_{ij} = \frac{c_{ij}}{k_i - 1 + k_j - 1 - c_{ij}}, \quad (8)$$

where  $c_{ij}$  is the number of common friends between node  $i$  and  $j$ ,  $k_i$  and  $k_j$  is the degree of  $i$  and  $j$ , respectively. Based on the definition of tie strength, we can also define the strength of a node as the averaged strength of all the ties connected to it. It is

$$w_i = \frac{\sum_{j \in \{i's \text{ neighbors}\}} w_{ij}}{k_i}. \quad (9)$$

Then we can define the averaged strength of the nodes with degree  $k$  as

$$w(k) = \frac{\sum_{\{i \in V | k_i = k\}} w_i}{|\{i \in V | k_i = k\}|}. \quad (10)$$

## 4. A NEW MAGIC NUMBER

In this section, we start from some observations on the local measures of a sampled online social network from Facebook. By coupling the variation of local measures with the increment of the degree, we discover an interesting phenomenon. Then we find this phenomenon pervasively exists in other online social networks. We summarize these observations to formulate our conjectures. In the end of this section, we also validate them by investigating the real trace of online interactions between users.

### 4.1 The turning point on local measures

The sample dataset we use comes from [23] and it is publicly available. This dataset is a snapshot of Facebook network in the city of New Orleans, so we denote it as **NewOrleans**. It contains 63292 nodes and 816886 ties. Its  $k_{max} = 1098$ ,  $k_{min} = 1$ ,  $\langle k \rangle = 25.8$  and  $C = 0.22$ .

We firstly analyze the measurement of degree distribution for the **NewOrleans** network. As shown in Figure 1, we surprisingly find a gentle slope in the interval between [0,200] in the degree distribution. Unlike a straight line in typical power-law, an turning point obviously appears and power-law only exists in the tail.

We calculate the fraction of users in the gentle slope interval and find more than 94% users are in it. Why most users are so "crowded" in this narrow interval while the number of users begin to drop dramatically beyond the interval?

In fact, similar degree distribution has already been observed by Golder et al. in [7]. The turning point in their dataset approaches 250, and they argue that it is because

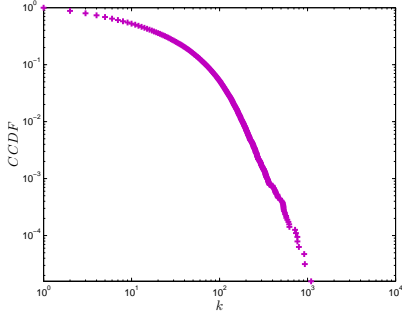


Figure 1: Degree distribution of NewOrleans

friendship in **Facebook** cannot completely represent conventional friendship. Nevertheless, they did not further explore the problem to give a more detailed explanation.

To sum up, node degree  $k \in [200, 300]$  denote a threshold value, and distributions are different in the two sides of it. By revealing the threshold, we may want to know whether there are any hidden facts lay behind it. To recall Dunbar's number mentioned in Section 2.1, we can see the threshold is not far from Dunbar's number 150. So does Dunbar's number play a vital role in this phenomenon? Does it exist or shift to a new magic number in online social networks? To provide more concrete evidences to explain the phenomenon, we observe how  $C(k)$ ,  $k_{nn}(k)$  and  $w(k)$  related with  $k$  in this network to see whether turning point also appears in these measurements, and we conclude our observations and remarks as follows.

**Observation 1.** We plot the distribution of clustering coefficient, as shown in Figure 2(a).  $C(k)$  steadily decreases with the increment of  $k$  at first, however, when  $k$  exceeds a certain threshold in the region of  $[200, 300]$ , the speed of decrement apparently raises. Therefore the turning point also exists in  $C(k)$  with almost the same value in the degree distribution. Here we simply denote it as  $k_T$  and  $k_T \in [200, 300]$ .

**Remark 1.** Clustering coefficient reflects the connections among neighbors of a node. High clustering coefficient indicates tightly connected neighborhood. In view of **Observation 1**, we can conclude that users with lower degrees ( $k < k_T$ ) have a well connected neighborhood, *i.e.*, quite a large fraction of these users' friends are acquainted with each other. It's not hard to explain this in the real social networks, as a person always get acquainted with some strangers through one of his friends. The behavior of meeting friends' friends is even strengthened in online social networks since it enables users to meet others with no restriction in time and space. For instance, Facebook users will receive notifications in their *News Feed* when their friends establish friendship with another user, and they can click "add as friend" to become friends as well. Moreover, almost all the social sites provide the feature of *Common Friends*, which lists the other users having common friends with you. It is this form of friends making mechanisms that leads to a denser network for lower-degree users. On the contrary, things cannot hold true for the users with high degrees. The averaged clustering coefficient drops to a very low point, meaning that although these users have hundreds of friends, they do not know them well indeed. In consequence, we

believe that most friends of these users seldom make new friends by way of them, making them have loose neighborhoods. It seems that users are divided into two types by the turning point  $k_T$ , with one type of users positioned in an acquaintance network and maintaining some meaningful connections, and the other type of users keeping some formalized relationships.

**Observation 2.** Figure 2(b) displays the distribution of degree correlations, the trend of which can represent the assortativity of the network. It's interesting to find there is a positive trend within  $k_T$ , and then  $k_{nn}(k)$  scatters with a slight negative trend beyond that. The same trend of  $k_{nn}(k)$  has also been observed in Mixi, an online social site in Japan [25].

**Remark 2.** The positive trend of  $k_{nn}(k)$  at the first stage is consistent with the assortativity property of the offline social networks [19]. In this stage, nodes tend to connect to those with similar degrees. However, the negative scattering in the range  $[200, 300]$  shows that the network transforms to a disassortative network. That is to say, for the users with degrees higher than  $k_T$ , they are not preferentially connected to the nodes with similar degrees. Results in **Observation 2** suggest that users on the two sides of the turning point behave differently in establishing friendships. On the left side, users are prone to connect other users with similar number of friends, while users on the right side may randomly add large amount of friends without much consideration. The results from degree correlations provide another evidence to prove the distinction of users in online social networks.

**Observation 3.** It is easy to find that there exists an turning point in Figure 2(c), still within the value  $k_T$ . Before reaching the turning point,  $w(k)$  steadily remains a high value with slight increment as  $k$  grows, but it begins to decrease when  $k$  is higher than  $k_T$ .

**Remark 3.** The previous work [12] has shown that in social networks, like mobile communication network, the tie strength  $w_{ij}$  between node  $i$  and  $j$  is strongly related with the frequency of interaction between users. It is also found that in online social networks, nodes with more mutual friends tend to trust each other and share more similar interests [17]. So the averaged tie strength  $w_i$  of user  $i$  can imply the overall quality of relationships with friends to some extent. View from **Observation 3**, users with degrees lower than  $k_T$  keep a high tie strength value, suggesting that these users maintain friendships with high quality. In contrast, for users with degrees higher than  $k_T$ , their strength is weak and become even weaker with the increment of  $k$ . Then we infer that among their friendships, some are fragile and not trusted. In the online social sites, this situation possibly corresponds to the following scenarios:

- High-degree users are really popular to attract a lot of low-degree nodes to add them as friends. For example, they can be a movie star, a notable scientist or even a famous enterprise, etc. However, they do not communicate with those "new" friends frequently. Therefore the ties between them become weak and some may even vanish eventually.
- It's human nature to pursue for prestige in the society, while the forms may be different. Some people are eager to gain popularity by randomly sending thousands of invitations to be added as friends in online social networks. They can probably acquire thousands of online friends as time goes by, however, most of

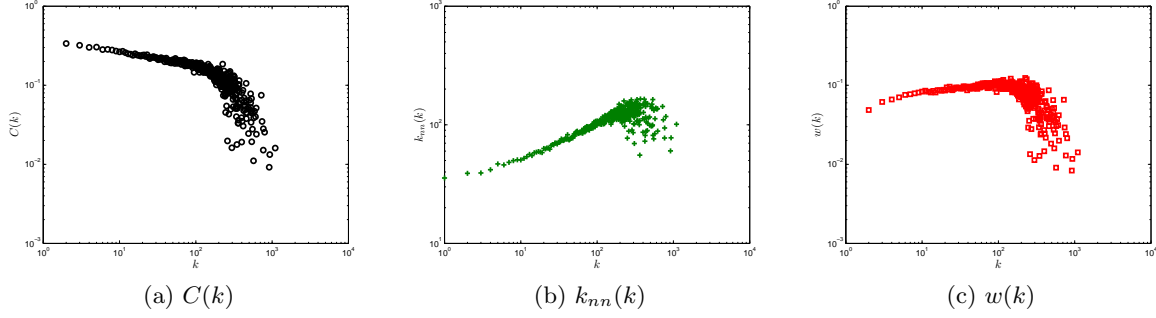


Figure 2: The variations of local measures for NewOrleans.

the relationships are of no meaning and the number of shared friends is certainly quite small.

We surprisingly find that the same turning point appears in all the measurements, coinciding with what we’ve found in the degree distribution. Based on the above observations and remarks, we can conjecture that in the dataset we used, there does exist a threshold  $k_T \in [200, 300]$ . As a matter of fact, this magic threshold distinguishes users by their variations of the local topological properties in online social networks. Furthermore, we have found that users’ online behaviors can be characterized by these properties.

As Dunbar’s number implies, an individual could not maintain more than 150 friendships in the real world effectively because of the time and cognitive constraints. We unveil in the previous observations that users beyond the turning point behave quite differently. They keep loose neighborhood with only a few friends knowing each other, and they randomly connect to users of different degrees or demographics with no preference; more importantly, the weak averaged tie strength of these users indicates poor relationships with their friends. It seems that the turning point  $k_T$  plays a similar role in online social networks as Dunbar’s number in the real society due to these facts. Although it is at low cost to make new friends and maintain friendships in online social networks, the number of friendships one can handle is still limited. In view of this, we infer that the turning point  $k_T$  is just the upper limit of well maintained online friendships, and users can be divided into two categories based on this point.

Online social networks have gained so much popularity in the worldwide. However, people’s attitude towards it gradually changes with more intensive use. In this day and age, logging in your Facebook account is not purely for entertainment but transforms into a habit and everyday life, just like checking out your emails or browsing a web page. Many users become rational in using social sites, as they carefully maintain a well connected neighborhood, most of which are “cloned” from their offline social networks. So the motives of using online social networks for the so called “rational users” become as simple as maintaining friendships. They stick to this new form of friendship maintenance because it shortens the distance between friends, as they can be informed of what is happening to their friends through the news feed with no geographic constraints. In fact, “rational users” corresponds to the users within  $k_T$ . Nonetheless, users beyond the threshold denote another type of users. We define them as “aggressive users” as they aggressively accu-

Table 1: Datasets

| Dataset     | $ V $   | $ E $    | $k_{max}$ | $\langle k \rangle$ | $C$  |
|-------------|---------|----------|-----------|---------------------|------|
| Georgetown  | 9388    | 425619   | 1235      | 90.67               | 0.22 |
| Oklahoma    | 17420   | 892524   | 2568      | 102.47              | 0.23 |
| Princeton   | 6575    | 293307   | 628       | 89.22               | 0.24 |
| UNC         | 18158   | 766796   | 3795      | 84.46               | 0.20 |
| Livejournal | 5203764 | 48709773 | 15017     | 18.72               | 0.27 |

mulate large amount of friends while most relationships are inactive and lack of interactions.

However, we draw our conclusions only from one dataset of Facebook so far. Does the magic threshold exist in other online social network samples? Or if it exists, do users behave distinctively on the two sides of the threshold? In the next section, we employ more datasets, either larger or smaller, to further discuss these issues.

## 4.2 Pervasiveness

In this section, in order to prove the ubiquity of the phenomena we found above, we import another five datasets of online social networks. The first four datasets are provided by [21] and are all publicly available. The four datasets are the complete Facebook networks whose ties are within four American universities. The four universities are Georgetown University(Georgetown), Princeton University(Princeton), University of Oklahoma(Oklahoma) and University of North Carolina at Chapel Hill(UNC), respectively. The fifth data set was collected from Livejournal, denoted as Livejournal. It is also public to the research community[17]. The detailed descriptions of these datasets are listed in Table 1.

Next, we perform the same measurements on these five datasets as on NewOrleans. Just as shown in Figure 3, for all the measures, including  $CCDF$ ,  $C(k)$ ,  $k_{nn}(k)$  and  $w(k)$ , there still exists a threshold  $k_T \in [200, 300]$ , which is independent to  $|V|$ . Especially, in the dataset of Livejournal, the size of the network is as many as five millions, however, the  $k_T$  is still in the range between 200 and 300.

In addition, as users’ demographic data is provided in the datasets of Georgetown, Oklahoma, Princeton and UNC, we can conduct experiment to examine the homophily[16] property in the network. In these datasets, we investigate the homophily in the following contexts, including student or faculty flag, gender, major, second major, dorm, year of enrolling and high school. The attribute vector for a node can be defined as  $Hi(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i, a_7^i)$ , where  $a_l^i (l = 1, 2, \dots, 7)$  corresponds to the above properties se-

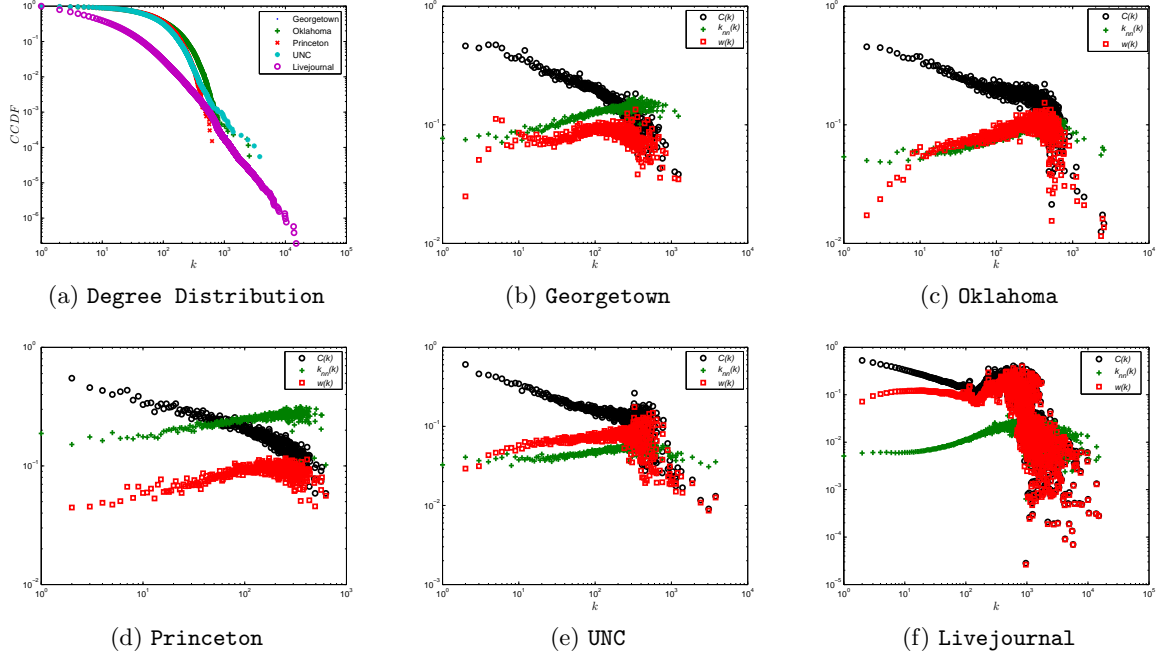


Figure 3: Results from other datasets.

quentially. we define binary distance for each attribute , which means,  $\|a_l^i - a_l^j\| = 1$  when  $a_l^i \neq a_l^j$ , otherwise,  $\|a_l^i - a_l^j\| = 0$ . Then the homophily distance between node  $i$  and  $j$  can be defined simply as

$$d_{ij} = \|H_i - H_j\|_2 = \sqrt{\sum_{l=1}^7 \|a_l^i - a_l^j\|^2}. \quad (11)$$

The averaged homophily distance for node  $i$  can be defined as

$$d_i = \frac{\sum_{j \in \{i's \text{ neighbors}\}} d_{ij}}{k_i}, \quad (12)$$

where  $k_i$  is degree of  $i$ . Then the averaged homophily distance for the nodes with degree  $k$  can be defined as

$$H(k) = \frac{\sum_{\{i \in V | k_i = k\}} d_i}{|\{i \in V | k_i = k\}|}. \quad (13)$$

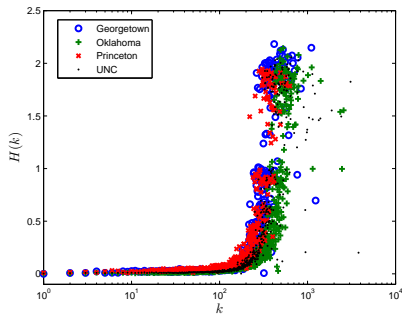


Figure 4: Homophily property.

As shown in Figure 4, there is an explosive increment of  $H(k)$  beyond the same degree value  $k_T$ . As  $H(k)$  measures

the similarity between users' attributes, it convinces our inference that "rational users" have a tendency to establish friendships with their familiar friends in the real world, usually sharing some common demographics such as dorm, major etc. While "aggressive users" are somewhat aimless to add as many friends as possible, so the demographics vary a lot.

**Remark** We confirm that our findings from the dataset of **NewOrleans** are pervasively existing in online social networks through the above observations. We even strengthen our conjectures that many connections of "aggressive users" are established with no substantial meaning by observing the homophily property. That is to say, threshold phenomenon is not a special feature of the sampled dataset, but a general character of online social networks.

All the previous analysis, based on the six datasets, mainly relies on the topological feature of the networks. However, pure structural information cannot be so convinced to represent the interaction between users. So it is necessary to validate our former conjectures through some real-world traces of interactions, which will be introduced in the next subsection.

### 4.3 Validation

Generally, the interaction data flowing in the online social sites is hard to collect. Because there are always some configurations of privacy protection in these sites. To our best knowledge, we collect two datasets from [23] and [18], respectively. Both of the datasets are publicly available and anonymized for research purpose. Besides, the two datasets both come from Facebook.

The first dataset was collected from the Facebook network in the city of New Orleans, which was related to the dataset of **NewOrleans** mentioned in the previous sections. They collected the publicly accessible profile pages and ab-

stracted the list of the “Wall” post. So we denote this dataset as **NewOrleans-Wall**. “Wall” is a popular feature of Facebook, through which a user can leave messages on his friends’ profile pages and the friends can also reply him by leaving messages, too. It is a classical and easy interacting way in Facebook. The dataset covers as many as 46952 users.

The second dataset was collected through some Facebook applications developed by the authors of [18], denoted as **Facebook-Applications**. They developed three popular Facebook applications named as GotLove?, HUG and Fighters’ Club, respectively. The authors collected about 3-week traces starting from March 20, 2008. Here we only use the data from GotLove? and HUG. In GotLove?, one node can send ‘love’ to its friends. And in HUG, a user A can send a virtual ‘hug’ to a friend B. The dataset from GotLove? contains 642088 active users and the one from HUG contains 198379 active users.

Based on these traces of interaction in Facebook, we try to relate the degree of nodes with their activity strength and then to validate our previous findings. In the dataset of **NewOrleans-Wall**, we define the node  $i$ ’s activity strength as the length of its list of the wall post, which can be denoted as  $L_i$ . The longer the list of the wall post is, the more interactions between  $i$  and its neighbors happen. Then the averaged activity strength of the nodes with degree  $k$  can be defined as

$$L(k) = \frac{\sum_{\{i \in V | k_i = k\}} L_i}{|\{i \in V | k_i = k\}|}. \quad (14)$$

For the dataset of **Facebook-Applications**, we simply extract the number of ‘love’ or ‘hug’ the node  $i$  has sent ( $s_i$ ) and received ( $\gamma_i$ ). Then the reciprocation of a node  $i$ , denoted as  $r_i$ , can be defined as the ratio of  $\gamma_i$  and  $s_i$ , *i.e.*,  $r_i = \gamma_i/s_i$ . Then the averaged reciprocation of the nodes with degree  $k$  can be defined as

$$r(k) = \frac{\sum_{\{i \in V | k_i = k\}} r_i}{|\{i \in V | k_i = k\}|}. \quad (15)$$

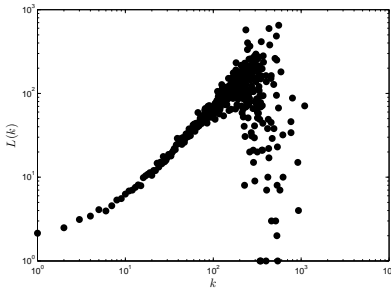


Figure 5: **NewOrleans-Wall**

**Observation 1.** Just as shown in Figure 5, as  $k$  increases,  $L(k)$  increases quickly. However, when  $k$  reaches out of the range  $[200, 300]$ ,  $L(k)$  stops increasing and begin to fluctuate. This is consistent with our former finding from the pure topological data. The fluctuation of  $L(k)$  implies that some of the users with degrees higher than the threshold  $k_T$  have shorter list of the wall post and their interaction remains in a rather low level.

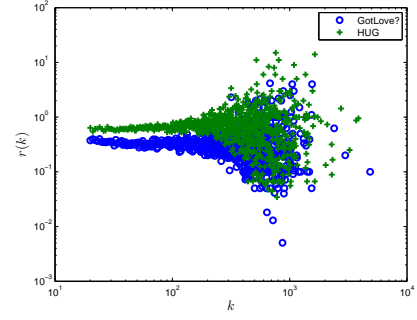


Figure 6: **Reciprocation varies with  $k$ .**

**Observation 2.** It is obviously shown in Figure 6 that the reciprocation of the nodes approaches 1 when  $k < k_T$ . It means almost all the lower-degree users’ sent ‘love’ and ‘hug’ are reciprocated. However  $r(k)$  diverges when  $k > k_T$ , indicating that their interaction with friends is not symmetric. As has been illustrated in Section 4.1, users accumulate many friends either because of popularity or eagerness for prestige, thus we can infer that some are far below 1 since these users’ behaviors are ignored by their friends, while some are above 1 because they are popular enough to receive ‘love’ and ‘hug’ from many fans.

**Remark** The above experiments further validate our conjecture that there exists a degree threshold in online social networks. If a user’s degree is higher than the threshold, the user then cannot maintain all its online friendships well and part of the friendships can be easily ignored by the friends on the another end.

**Summary** Until now, by validating from the real-world traces of user interactions online, we can reasonably conclude that there still exists an upper limit on the number of the friendships in online social networks as Dunbar’s number in offline social networks. If users have more friends than the limit, it is impossible for them to treat each tie equally. Because of this, extraordinary dynamics will be bred when the degree goes up to the limit. As a result, we see the phenomenon that high-degree users keep overall relationships of low quality. In a further step, we believe that users with mediate degrees are “rational users” with the motivation to maintain old friendships while users who have friends more than the limit are likely to be “aggressive users” seeking for new friends always.

However, little attention has been paid to these phenomena in many current models, and they could not interpret the generation rule completely. So we aim to understand how it generates by presenting a new model in the next section.

## 5. A NEW MODEL

In this section, we present a new model to interpret the generation of the upper limit found in the previous sections. We start from summarizing users’ online behaviors from previous observations and conclusions. Next we introduce an inspiring model, and point out the imperfections to apply the model to the situation of online social networks. Then we incorporate the characters of users’ online behaviors to propose the new model. At last we examine the properties of our simulated network to compare with the real networks.

As has been discussed in Section 4, users’ online friends

adding follows these rules:

1. When users first register in the sites, they tend to search for their offline acquaintances, and the network system would also recommend some friends based on the user's profiles. These friends become their initial online contacts, and they provide the basis for further friend making.
2. Even more conveniently than that in the real world, users can set up connections with their friends' friends simply by viewing the friends list and choosing who they already know to add as friends. Besides, some sites also recommend friends' friends to help users find more friends online.
3. As is illustrated in previous sections, most users are "rational users" to be trapped in a magic number circle. After the number of friends is accumulated to a certain upper limit, they would stop adding more friends; or what's worse, they may even reject others' invitation to be added as friends. While only a few of sociable users jump out of the circle to become "aggressive users", they'd like to add as many friends as possible actively. In fact, this process results in random linkage since the "aggressive users" do not have explicit target users to link to.
4. Though online friendship maintenance costs almost no money or time, "unfriend" situation still exists. For example, Section 4.1 implies that the ties between "aggressive users" and their friends are fragile and can vanish in some way. Moreover, social sites like Facebook will "pull" all friends' updates to the users' news feed; however, some users may be annoyed by continually receiving one specific friend's message, thus "unfriend" happens under this circumstance and the links are removed.

In order to model the growth of social networks, Jin et al. [13] proposed two models based on three general principles. First, the individuals tend to meet with those ones who have one or more common friends with them. Second, acquaintances between the individuals who rarely meet decay over time, which means some ties may vanish. Third, there is a maximum degree limit for an individual. However, features of the online social network are different from their assumptions. For instance, the online social networks usually starts to evolve from a real-world social network. The site will urge the user to invite their real-world friends to the site or provide easy way to search them in the site. The another difference is the limit of the degree. In their models, a node can not have a higher degree than the maximum degree. But in online social networks, the maximum value set by the site may be high, like 5000 in Facebook or even more. Because of this, their model can only control the maximum degree of the nodes, but can not interpret the threshold degree as we find. In addition, this is also worth to be noticed that in their model, the constraints of time and cognition are only associated with the control of maximum degree. Given these imperfections, we try to model the online social network based on the following assumptions:

- The network start to evolve from a existing social network. Here, we simply start it from a BA network. The network evolves only by adding or removing ties.

- New ties between nodes with common friends are preferred. However, for the nodes with degrees higher than a threshold, the probability of tie be established between its neighbors is lower.
- Some nodes may search and add a random node as a friend.
- Some ties may vanish, especially for the nodes with high degrees.

Guided by these principles, we present a simple model, called *BA - shift* as follows:

- Step 1: load a BA network, denoted as  $BA(V, E_0)$ , where  $V$  is the set of nodes and  $E_0$  is the set of original ties.
- Step 2: In each time unit, perform the following actions:

- Action 1: Select a node  $i$  with the probability

$$p(i) = \frac{k_i(k_i - 1)f(k_i)}{\sum_{j \in V} k_j(k_j - 1)f(k_j)}, \quad (16)$$

where  $k_i$  is the current degree of  $i$ . Here we use  $f(k_i)$  to constrain the nodes with higher degrees than the threshold  $k_T$ . We define

$$f(k_i) = \frac{1}{e^{\beta(k_i - k_T)} + 1}, \quad (17)$$

where  $\beta$  is a parameter to control the extent of the constraint. If  $k_i > 2$ , randomly select two of its neighbors and establish a tie between them if they are not connected in the earlier stage. Repeat this action for

$$\frac{1}{2}c \sum_{i \in V} k_i(k_i - 1) \quad (18)$$

times, where  $c$  is the speed of adding new ties.

- Action 2: Select a node  $q$  with probability

$$p(q) = \frac{k_q + 1}{\sum_{j \in V} (k_j + 1)}. \quad (19)$$

If  $k_q > 1$ , select one of its neighbors randomly and remove the tie between them. Repeat this action for

$$\frac{1}{2}d \sum_{i \in V} k_i \quad (20)$$

times, where  $d$  is the speed of removing ties.

- Action 3: Randomly select a pair of nodes and add a tie between them if they are not connected originally. Repeat this action for  $|V|r$  times, where  $r$  is the speed of adding linkage randomly.

- Step 3: If the current averaged degree of the network reaches  $\langle k \rangle_{max}$ , stop evolving and return the network. Otherwise, increase the evolving time and then jump to Step 2.

**Remark 1.** In the first step we choose to load a BA network just because it is classical and simple. Many real-world networks are found scale-free, including the social networks.

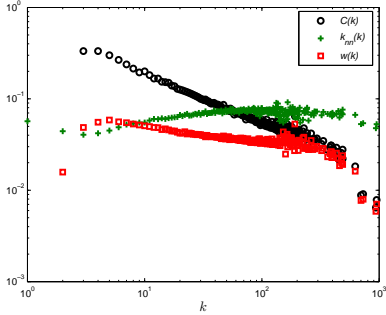
**Remark 2.** In the Step 2 Action 1, the nodes with higher degrees will be selected to increase the closeness of the network among their friends. However, when the node's degree

is higher than the threshold  $k_T$ , the probability of getting selected will decrease sharply. It responds to the situation that the nodes whose degrees have exceeded the threshold, some of their online friendships would be weak and it is hard for their neighbors to get acquainted through them. This is the essential part of the model, which is different from the others.

**Remark 3.** In the Step 2 Action 2, the nodes with higher degrees will be selected easily to lose a random acquaintance. Because for the high-degree nodes, it is easy to ignore some friendship for the constraints of time and cognition.

**Remark 4.** In the Step 2 Action 3, a pair of nodes will be randomly selected and connected. It responds to the phenomenon that some friendships are established casually in online social sites. For instance, some users may search other strange ones with common interests or just accept some unknown invitations.

In the following simulations, we denote the network generated by the model as  $BA - shift(|V|, \langle k \rangle_{max}, c, d, r, \beta, k_T)$ . The BA network we use contains 20000 nodes and 39973 ties originally. As showed in Figure 7, it is easy to find for all the local measures, including  $C(k)$ ,  $k_{nn}(k)$  and  $w(k)$ , there exists a turning point near  $k_T = 200$ . This result is consistent with the real-world datasets. However, for the BA network, the variations of  $C(k)$  and  $k_{nn}(k)$  keep decreasing steadily with  $k$ , while  $w(k)$  just increases without any descending tendency. In fact, compared with other models, such as JGN and BA,  $BA - shift$  pays more attention to understanding how the constraints of time and cognition affect the evolution of online social networks. The aim of the model is to unveil the generation of the threshold we find.



**Figure 7:**  $BA - shift(20000, 20, 0.0005, 0.0005, 0.0001, 8, 200)$

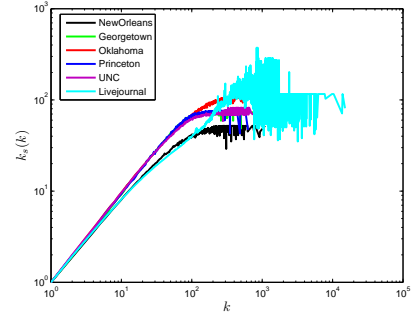
**Summary** In this section, we present a simple model to interpret the generation of the upper limit. Compared to Dunbar’s number, the value of the limit in online social networks is greater. We believe it will bring some impact and insight to the current situation.

## 6. BUSINESS INSIGHTS

### 6.1 Online viral marketing

Thanks to the thorough growth of online social networks in the recent decade, a new strategy for marketing has been deployed. Nowadays we may often see some comments on a particular product from our Facebook friends’ wall, or advertisements may appear in the form of tweets from the people we follow on Twitter. That’s indeed an instance of the online

viral marketing, as product information spreads from person to person directly (word-of-mouth) within the networks and influences people’s purchasing decisions. Viral marketing in online social networks may be quite effective as people may seriously consider friends’ recommendations. However, questions still remain on how to do it and where to start. Leskovec et al. [15] suggest that unlike epidemic spreading models, high-degree nodes are not so influential in viral marketing situation. This conclusion can be well supported by our observations, since “aggressive users” with thousands of friends only interact with a small group of friends. We validate this in a further step by importing the measurement of  $k_s(k)$ . As shown in Figure 8, the averaged k-shell value stops increasing and remains stable with slight fluctuations after the threshold, meaning that the core effect is not obvious for high-degree nodes. Just as illustrated in [14], some high-degree users are wrapped by large amount of low-degree users in the periphery, so that themselves are also positioned in the periphery and play a trivial role in spreading product information.



**Figure 8:** Variations of  $k_s(k)$  with  $k$ .

From what we have discussed in previous sections, we believe that due to the loosely connected neighborhood and lacking of interactions, the messages sent out by the “aggressive users” may be ignored easily. In contrast, “rational users” are tightly linked to each other, and these users’ online friends are largely covered by their offline friends. Therefore they can be more trustful, and their messages would be thought highly of by their friends. In consequence, their friends may be induced by their purchasing suggestions and behaviors.

### 6.2 Privacy management in online social networks

In order to guarantee a more authentic social network, most online social sites require users to provide their authentic personal information when firstly register in the sites. However, this can arouse the concerns for users’ privacy issues as users’ profile information such as ZIP code, gender and birthday may be stolen for improper use [9].

At the same time, users have begun to recognize the necessities of privacy protection in online social networks, especially the “rational users”. Since these users have “moved” quite a number of offline friends to the social sites, they regard the social sites as personal space to interact with their friends, so that the interaction can be quite private, even secret. In view of this, current social sites such as Facebook has already started to provide the service of privacy settings.

Users themselves can determine whether to reveal their information only to friends or to the public. In fact, friendships are regarded as binary in this situation, that is to say, all the private settings are equally effective to each friendship. However, as shown in our discussions, users cannot treat each tie equally. “rational users” indeed have different attitudes toward their online friends, so they may desire for a more detailed and flexible mechanism that enables them to have different privacy settings for different groups of friends.

However, things are different for “aggressive users”. They do not care too much about privacy, and instead they are willing to disclose their information to more users in order to gain popularity. Another particular phenomenon should be noted is that there exist “spammers” in online social networks. “Spammers” disguise as “aggressive users”, usually with fake profiles of celebrities, to establish so many connections with the intention to carry out identity theft [10], which is a great threat to online users. To detect such “spammers”, we can examine its interaction records because they only add friends but do not interact at all.

## 7. CONCLUSION

Just as unveiled in social networks, there is still a magic upper limit on users’ number of friendships that they can effectively maintain in online social networks. Through abundant experiments and validations, we conclude that users with considerable circles of friends within the magic number are “rational users”. They mainly use online social networks on the purpose of maintaining their old friendships. In contrast, “aggressive users” reach out of the magic number with the desire to make as many new friends as possible. We also propose a new online social network model to further explain users’ online behaviors. We think the findings of the new magic number and distinction of users are helpful in viral marketing and privacy management issues in online social networks.

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